A Sieve Bootstrap Test for Cointegration in a Conditional Error Correction Model - Additional simulations

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June 4, 2008

1 Introduction

This document contains additional simulation studies that investigate the sensitivity and robustness of the tests considered in the simulation studies in the paper.

A major difference with the simulations contained in the paper is that the simulations in this document were all performed using the so-called Warp-Speed Bootstrap (WSB) of Giacomini, Politis, and White (2007). This method greatly reduces the computational cost of performing the simulations. Giacomini et al. (2007) show that under quite general conditions the WSB is capable of calibrating the finite-sample coverage of bootstrap confidence intervals if the bootstrap is asymptotically valid (which we show in the paper). Because of the close relation between confidence intervals and hypothesis testing the method should work properly in our setting as well.

The remainder of this section examines the performance of the WSB more closely by making a comparison between the simulations performed in the paper and simulations for exactly the same set of DGPs evaluated using the WSB. Section 2, the main part of this document, contains extensions of the simulation study undertaken in Section 4 of the paper, i.e. without deterministic components. Section 3 is devoted to simulations belonging to Section 5 of the paper, i.e. with deterministic components.

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We thank Hans Manner and anonymous referees for helpful suggestions and comments concerning the simulation studies described in this document. The usual disclaimer applies.

1.1 No deterministic components

Our original setup looked as follows. We let the bivariate series $(y_t, x_t)'$ be generated from the triangular system

$$\begin{aligned} y_t &= \gamma x_t + u_t, \\ u_t &= \rho u_{t-1} + v_{1t}, \\ \Delta x_t &= v_{2t}. \end{aligned} \tag{1}$$

We take $\rho = 1$ to analyze the size of the tests, and $\rho < 1$ for the power. As we are interested in local power, we set $\rho = 1 + c/n$, where for the sample size we take $n = 50$ and $n = 100$. The tests are invariant to the true value of γ as long as it is non-zero, therefore we set $\gamma = 1$. Furthermore we set $u_0 = x_0 = 0$.

We generate the errors $v_t = (v_{1t}, v_{2t})'$ as

$$(1 - \Phi L)v_t = (1 + \Theta L)\varepsilon_t, \tag{2}$$

where ε_t is generated from a bivariate normal distribution with covariance matrix

$$\Sigma = \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}.$$

We select lag lengths by BIC, with maximum lag lengths of 8 for $n = 50$ and 11 for $n = 100$. Our results are based on 2000 simulations, but contrary to the paper (where we used 999 bootstrap replications per simulation), we employ the Warp-Speed Bootstrap here.

To compare how the WSB relates to our original simulations, we now give here the results for the original parametrization but evaluating using the WSB. We also test whether the corresponding rejection frequencies are statistically significantly different from each other using the Z-test for testing the homogeneity of two independent binomial proportions. 1, 2 or 3 stars behind a number indicate a significant difference at 10%, 5% and 1% respectively.

The simulation results are given in Table 1 to 3. As can be seen from the tables, there are some significant differences between the WSB and the original simulations. Note however that while those results are statistically significant, they are are qualitatively the same (we would not draw other conclusions from the WSB than we did in the paper).

Table 1: No serial correlation

r	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
				$n = 50$		
0	0	0.048	0.045	0.046	0.041	0.079
	-5	0.099	0.087**	0.106	0.107	0.165
	-10	0.293	0.301	0.279	0.232***	0.412
	-20	0.838	0.821	0.850*	0.851	0.935**
$\sqrt{0.3}$	0	0.042	0.042	0.039**	0.043	0.070*
	-5	0.179	0.157	0.178	0.186**	0.253
	-10	0.470***	0.556**	0.461***	0.499	0.630**
	-20	0.938	0.939***	0.939	0.948*	0.975**
$\sqrt{0.7}$	0	0.059	0.045	0.053	0.057	0.079
	-5	0.496	0.478*	0.486	0.475*	0.612
	-10	0.882	0.910	0.919**	0.933***	0.950**
	-20	0.995***	0.997	0.995***	0.997	0.999
				$n = 100$		
0	0	0.049	0.050	0.047	0.042	0.059
	-5	0.093	0.099	0.101	0.120	0.136
	-10	0.306	0.348*	0.325	0.290*	0.374
	-20	0.860	0.867	0.856	0.892***	0.905
$\sqrt{0.3}$	0	0.052	0.051	0.055	0.051	0.063
	-5	0.212**	0.162**	0.180	0.180	0.220
	-10	0.525	0.530	0.529	0.542	0.579
	-20	0.958	0.958	0.966	0.965	0.977
$\sqrt{0.7}$	0	0.054	0.051	0.055	0.047	0.066
	-5	0.561	0.542	0.538	0.537	0.600
	-10	0.920*	0.923	0.923*	0.921	0.941
	-20	1.000	1.000	1.000	0.999*	1.000

Table 2: Size

Φ	Θ	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.048*	0.059	0.052	0.059	0.100
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.058	0.051	0.052	0.042	0.167
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.055	0.059***	0.055	0.069**	0.220
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.057	0.058	0.051*	0.050	0.091
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.050	0.035***	0.036**	0.036***	0.182
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.068	0.079	0.067	0.068***	0.202
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.429*	0.621	0.373***	0.608	0.682
			$n = 100$			
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.058	0.051**	0.065	0.058	0.084
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.049	0.052	0.040	0.051	0.085
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.060	0.059	0.064	0.059	0.105
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.054	0.063	0.056	0.055	0.076
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.058	0.052**	0.056	0.054	0.103*
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.073	0.079	0.075	0.073	0.135
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.478	0.547*	0.493	0.514	0.615

Table 3: Power

dynamics	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.617	0.586***	0.596*	0.545	0.832
	-10	0.816***	0.782***	0.783***	0.775***	0.957
	-20	0.878***	0.957***	0.924	0.969*	0.995
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.352***	0.324	0.383*	0.303	0.599
	-10	0.769***	0.699***	0.730***	0.685***	0.931
	-20	0.958**	0.981***	0.933	0.974	0.996
			$n = 100$			
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.895	0.898	0.895	0.893	0.949
	-10	0.987*	0.991	0.985**	0.991	0.996
	-20	1.000	1.000	1.000	1.000	1.000
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.557***	0.517***	0.500	0.465	0.697
	-10	0.895	0.883	0.879***	0.882	0.968
	-20	0.999	0.999	0.999	0.999*	1.000

1.2 Deterministic components

The DGP used for the simulations with deterministic components corresponds to the DGP used before, except that we now add deterministic components to the triangular system as follows.

$$\begin{aligned} y_t &= \mu_1 + \tau_1 t + \gamma x_t + u_t, \\ u_t &= \rho u_{t-1} + v_{1t}, \\ \Delta(x_t - \mu_2 - \tau_2 t) &= v_{2t}. \end{aligned} \tag{3}$$

To keep the size of the experiment down we only report simulations for $n = 50$, and only for $c = 0$ and $c = -10$. Also, we only take three combinations of Φ and Θ: $\Phi = \Theta = 0$; $\Phi = \left[\begin{smallmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{smallmatrix}\right]$ and $\Theta = 0$; and $\Phi = 0$ and $\Theta = \left[\begin{smallmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{smallmatrix}\right]$. We restrict our attention to the two bootstrap variants $T^*_{v,n}$ and $T^*_{v,a}$ and the asymptotic test T_{as}.

We consider two models without a drift, and two where a drift is present. For the models without drift, a DGP with no deterministic components and one with just a constant term are chosen. For the models with drift, we select one DGP where the drift is canceled out in the direction of the cointegrating vector (i.e. $\tau_1 = 0$), and one where it is not. For each model we perform the tests with every deterministic specification that is appropriate for that specific model.

As in the no deterministics case, we report the results for the original simulations with the WSB is applied in Table 4. For the cases where there are short-run dynamics, we see that quite some rejection frequencies differ significantly from the corresponding rejection frequencies in the original tables. While the differences are clearly more pronounced than for the simulations without deterministic components, the conclusions that can be drawn form the new results still do not change. Besides, for the additional simulations we will perform with deterministic components the performance of the WSB relative to the original simulations is essentially irrelevant (as we will show in Section 3).

Table 4: Size and power for tests with deterministic trends

μ_1 τ_1 / μ_2 τ_2	$D_t^{(r)}$	$T^*_{v,n}$	$T^*_{v,a}$	T_{as}	$T^*_{v,n}$	$T^*_{v,a}$	T_{as}	$T^*_{v,n}$	$T^*_{v,a}$	T_{as}
		$\Phi = \Theta = 0$			$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$			$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$		
					$c = 0$					
	$D_t^r = 1$	0.046	0.056	0.110	0.057	0.066***	0.282	0.051	0.053***	0.234
0 0	$D_t = 1$	0.051	0.054	0.093	0.048	0.061***	0.231	0.059*	0.068***	0.230*
0 0	$D_t^r = t$	0.032**	0.035	0.125	0.045	0.070***	0.358	0.034***	0.069***	0.330
	$D_t' = 1, t$	0.056	0.056	0.129	0.045	0.057***	0.338	0.061	0.088***	0.333
	$D_t^r = 1$	0.055	0.052	0.105	0.047	0.064***	0.287	0.060	0.059***	0.245
1 0	$D_t = 1$	0.056	0.048	0.091	0.051	0.059***	0.260	0.051**	0.069***	0.231
1 0	$D_t^r = t$	0.046*	0.048	0.130	0.035	0.068***	0.372	0.046**	0.070***	0.329**
	$D_t' = 1, t$	0.047*	0.050**	0.133*	0.053***	0.057***	0.339	0.036***	0.053***	0.326
1 0	$D_t^r = t$	0.049	0.047	0.135*	0.029	0.069***	0.367	0.039***	0.061***	0.334
1 1	$D_t' = 1, t$	0.048	0.048	0.136	0.035	0.068***	0.346	0.040***	0.085***	0.343
1 1	$D_t^r = t$	0.050	0.049	0.143	0.044	0.069***	0.371	0.035***	0.054***	0.348
1 1	$D_t' = 1, t$	0.052	0.044	0.123	0.032	0.051***	0.350**	0.045***	0.048***	0.348**
					$c = -10$					
	$D_t^r = 1$	0.235	0.293***	0.434	0.574***	0.579**	0.894	0.584**	0.370***	0.802
0 0	$D_t = 1$	0.303	0.301	0.454	0.580***	0.591**	0.880	0.469***	0.371***	0.798
0 0	$D_t^r = t$	0.141**	0.143	0.367	0.382***	0.306***	0.835**	0.285***	0.135***	0.709
	$D_t' = 1, t$	0.169	0.171	0.385	0.378***	0.278***	0.836***	0.303***	0.186*	0.716
	$D_t^r = 1$	0.236	0.217**	0.427	0.585***	0.560***	0.900	0.514**	0.463	0.806
1 0	$D_t = 1$	0.255**	0.254**	0.443	0.544***	0.555***	0.881	0.513***	0.434	0.799
1 0	$D_t^r = t$	0.159**	0.136	0.365	0.431***	0.307***	0.833	0.336**	0.127***	0.734
	$D_t' = 1, t$	0.190***	0.171	0.388	0.370***	0.295*	0.830***	0.294***	0.196	0.718
1 0	$D_t^r = t$	0.163***	0.161*	0.356	0.423***	0.304**	0.846	0.312***	0.217**	0.737**
1 1	$D_t' = 1, t$	0.173	0.177	0.400	0.364***	0.234***	0.818	0.269***	0.178***	0.687**
1 1	$D_t^r = t$	0.173	0.171	0.368	0.374***	0.287***	0.853	0.324***	0.135***	0.707
1 1	$D_t' = 1, t$	0.154	0.167	0.401	0.393***	0.282**	0.809	0.271***	0.194	0.728**

2 Additional simulations without deterministic trends

In this section we will investigate the sensitivity and robustness to changes in our simulation setup. We will focus on the tests without deterministic components. Below we will discuss four types of changes to the simulation setup; the first three relate to the DGP, the last relates to the lag length selection.

2.1 GARCH errors

The first modification of the setup above is by letting the errors ε_t follow a multivariate GARCH process. Specifically, we let $\varepsilon_t = \Sigma_t^{1/2}\nu_t$, where ν_t is bivariate standard normal. We generate Σ_t as

$$\Sigma_t = \Sigma + \alpha\varepsilon_{t-1}\varepsilon_{t-1}' + \beta\Sigma_{t-1} \tag{4}$$

where Σ is the same as in the original experiment. Two sets of experiments were carried out: the first with $\alpha = 0.2$, $\beta = 0.7$, and the second with $\alpha = 0.7$, $\beta = 0.2$.

The reason to examine GARCH processes is that they do not belong to the class of linear processes, and therefore the sieve bootstrap is not valid, contrary to the block bootstrap for example. The values of the GARCH parameters were chosen such that the process is stationary, but close to an integrated GARCH process.

Results are presented in Table 5 to 7 for the case where $\alpha = 0.2$ and $\beta = 0.7$, and in Table 8 to 10 for the case where $\alpha = 0.7$ and $\beta = 0.2$. The performance of the bootstrap tests is fairly similar in the GARCH setting as in the i.i.d. setting. Therefore we can conclude that the sieve bootstrap tests are robust to this kind of nonlinearity.

Table 5: No serial correlation - GARCH: $\alpha = 0.2$, $\beta = 0.7$

r	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
0	0	0.059	0.059	0.067	0.064	0.103
	-5	0.134	0.121	0.136	0.116	0.197
	-10	0.320	0.323	0.327	0.309	0.454
	-20	0.870	0.818	0.806	0.829	0.913
$\sqrt{0.3}$	0	0.054	0.054	0.061	0.058	0.100
	-5	0.163	0.138	0.154	0.139	0.216
	-10	0.361	0.397	0.325	0.366	0.487
	-20	0.788	0.844	0.838	0.845	0.909
$\sqrt{0.7}$	0	0.070	0.064	0.059	0.071	0.106
	-5	0.169	0.149	0.153	0.171	0.233
	-10	0.379	0.331	0.364	0.386	0.515
	-20	0.864	0.871	0.859	0.892	0.936
			$n = 100$			
0	0	0.048	0.051	0.058	0.059	0.077
	-5	0.154	0.116	0.117	0.123	0.157
	-10	0.360	0.395	0.359	0.369	0.416
	-20	0.861	0.866	0.876	0.858	0.892
$\sqrt{0.3}$	0	0.060	0.056	0.053	0.064	0.081
	-5	0.136	0.154	0.142	0.143	0.175
	-10	0.350	0.378	0.387	0.365	0.439
	-20	0.895	0.884	0.868	0.864	0.906
$\sqrt{0.7}$	0	0.054	0.059	0.057	0.051	0.077
	-5	0.146	0.146	0.148	0.169	0.189
	-10	0.367	0.439	0.407	0.431	0.462
	-20	0.864	0.877	0.839	0.899	0.911

Table 6: Size - GARCH: $\alpha = 0.2$, $\beta = 0.7$

Φ	Θ	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
		$n = 50$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.063	0.072	0.073	0.064	0.113
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.067	0.068	0.070	0.057	0.195
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.070	0.061	0.052	0.055	0.179
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.050	0.057	0.053	0.059	0.097
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.050	0.043	0.052	0.041	0.184
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.053	0.061	0.054	0.054	0.165
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.422	0.597	0.395	0.582	0.667
		$n = 100$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.081	0.077	0.075	0.069	0.092
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.064	0.063	0.055	0.059	0.106
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.061	0.065	0.067	0.060	0.115
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.077	0.068	0.082	0.064	0.087
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.057	0.052	0.057	0.055	0.110
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.075	0.078	0.068	0.073	0.139
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.472	0.518	0.477	0.517	0.586

Table 7: Power - GARCH: $\alpha = 0.2$, $\beta = 0.7$

dynamics	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
	-5	0.270	0.261	0.268	0.239	0.509
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.455	0.419	0.449	0.381	0.703
	-20	0.575	0.642	0.557	0.626	0.811
	-5	0.149	0.139	0.177	0.147	0.388
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.404	0.446	0.376	0.386	0.739
	-20	0.722	0.812	0.717	0.820	0.938
			$n = 100$			
	-5	0.515	0.503	0.498	0.498	0.671
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.778	0.760	0.773	0.760	0.886
	-20	0.891	0.923	0.898	0.895	0.955
	-5	0.226	0.202	0.207	0.202	0.378
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.559	0.548	0.562	0.619	0.770
	-20	0.925	0.951	0.927	0.952	0.975

Table 8: No serial correlation - GARCH: $\alpha = 0.7$, $\beta = 0.2$

r	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
0	0	0.078	0.078	0.081	0.086	0.125
	-5	0.186	0.185	0.190	0.171	0.257
	-10	0.408	0.414	0.387	0.402	0.503
	-20	0.750	0.780	0.739	0.793	0.875
$\sqrt{0.3}$	0	0.080	0.074	0.076	0.074	0.125
	-5	0.193	0.183	0.185	0.188	0.256
	-10	0.385	0.432	0.392	0.432	0.522
	-20	0.803	0.802	0.815	0.810	0.872
$\sqrt{0.7}$	0	0.079	0.081	0.088	0.081	0.133
	-5	0.173	0.166	0.154	0.176	0.250
	-10	0.374	0.410	0.393	0.387	0.519
	-20	0.801	0.816	0.827	0.826	0.887
			$n = 100$			
0	0	0.097	0.109	0.108	0.104	0.118
	-5	0.156	0.155	0.180	0.169	0.206
	-10	0.411	0.392	0.393	0.398	0.456
	-20	0.822	0.822	0.822	0.816	0.858
$\sqrt{0.3}$	0	0.091	0.089	0.092	0.097	0.114
	-5	0.187	0.177	0.180	0.173	0.208
	-10	0.418	0.403	0.443	0.400	0.480
	-20	0.817	0.822	0.819	0.813	0.865
$\sqrt{0.7}$	0	0.082	0.088	0.100	0.085	0.110
	-5	0.184	0.172	0.171	0.180	0.222
	-10	0.395	0.397	0.384	0.427	0.468
	-20	0.811	0.838	0.822	0.842	0.861

Table 9: Size - GARCH: $\alpha = 0.7$, $\beta = 0.2$

Φ	Θ	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
		$n = 50$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.090	0.089	0.082	0.083	0.132
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.077	0.071	0.071	0.078	0.210
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.058	0.064	0.052	0.056	0.174
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.081	0.075	0.075	0.079	0.128
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.047	0.044	0.042	0.047	0.194
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.067	0.066	0.051	0.057	0.175
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.400	0.573	0.418	0.563	0.644
		$n = 100$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.066	0.068	0.068	0.070	0.092
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.066	0.064	0.064	0.064	0.112
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.054	0.067	0.056	0.067	0.116
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.080	0.069	0.080	0.086	0.106
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.058	0.056	0.057	0.065	0.120
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.083	0.087	0.068	0.072	0.138
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.446	0.513	0.454	0.489	0.579

Table 10: Power - GARCH: $\alpha = 0.7$, $\beta = 0.2$

dynamics	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
	-5	0.180	0.180	0.195	0.179	0.441
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.253	0.328	0.275	0.293	0.571
	-20	0.360	0.422	0.363	0.434	0.637
	-5	0.133	0.120	0.172	0.129	0.356
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.311	0.328	0.273	0.312	0.630
	-20	0.572	0.688	0.541	0.706	0.859
			$n = 100$			
	-5	0.326	0.350	0.348	0.367	0.521
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.555	0.555	0.576	0.594	0.733
	-20	0.687	0.741	0.705	0.731	0.813
	-5	0.207	0.183	0.161	0.171	0.339
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.445	0.457	0.464	0.470	0.666
	-20	0.825	0.852	0.801	0.845	0.919

2.2 Markov-switching dynamics

The next modification concerns the short-run dynamics as we let the short-run dynamics vary over time. In our simulation DGP we replace equation 2 by

$$(1 - \Phi_t L)v_t = (1 + \Theta_t L)\varepsilon_t. \tag{5}$$

We let Φ_t and Θ_t be generated as

$$\Phi_t = \begin{matrix} \Phi_1 & \text{if } S_t = 0 \\ \Phi_2 & \text{if } S_t = 1 \end{matrix} \qquad \Theta_t = \begin{matrix} \Theta_1 & \text{if } S_t = 0 \\ \Theta_2 & \text{if } S_t = 1 \end{matrix}$$

where S_t is a first-order two state Markov chain with transition probabilities

$$P = \begin{bmatrix} p_1 1 & 1 - p_1 1 \\ 1 - p_2 2 & p_2 2 \end{bmatrix}.$$

We always take Φ_1 and Θ_1 equal to the values of Φ and Θ from the original simulations. Φ_2 and Θ_2 are selected as

(i) If $\Phi_1 = \Theta_1 = 0$, then $\Phi_2 = \Theta_2 = \left[\begin{smallmatrix} -0.8 & 0 \\ 0 & -0.8 \end{smallmatrix}\right]$.

(ii) If $\Theta_1 = \left[\begin{smallmatrix} -0.8 & 0 \\ 0 & -0.8 \end{smallmatrix}\right]$, then $\Theta_1 = \left[\begin{smallmatrix} 0.2 & 0.5 \\ 0.2 & 0.5 \end{smallmatrix}\right]$.

(iii) Otherwise, $\Phi_2 = -\Phi_1$ and $\Theta_2 = -\Theta_1$.

Two sets of experiments were carried out: the first with relatively few switches ($p_{11} = 0.8$, $p_{22} = 0.7$), the second with relatively many switches ($p_{11} = 0.3$, $p_{22} = 0.2$).

Results for the first set of experiments are given in Table 11 to 13; results for the second set are given in Table 14 and 16. For both sets of experiments the bootstrap tests retain their good performance.

Table 11: $\Phi_1 = \Theta_1 = 0$ - MS: $p_{11} = 0.8$, $p_{22} = 0.7$

r	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
0	0	0.085	0.085	0.079	0.082	0.175
	-5	0.271	0.257	0.238	0.258	0.419
	-10	0.493	0.515	0.489	0.519	0.691
	-20	0.859	0.881	0.856	0.897	0.950
$\sqrt{0.3}$	0	0.073	0.077	0.071	0.083	0.167
	-5	0.221	0.207	0.221	0.221	0.435
	-10	0.526	0.498	0.472	0.527	0.787
	-20	0.900	0.945	0.918	0.937	0.983
$\sqrt{0.7}$	0	0.085	0.082	0.070	0.089	0.171
	-5	0.304	0.358	0.393	0.345	0.644
	-10	0.773	0.817	0.771	0.791	0.950
	-20	0.964	0.991	0.971	0.992	0.998
			$n = 100$			
0	0	0.058	0.063	0.062	0.067	0.092
	-5	0.141	0.137	0.148	0.129	0.200
	-10	0.380	0.329	0.348	0.326	0.467
	-20	0.852	0.753	0.899	0.799	0.921
$\sqrt{0.3}$	0	0.070	0.055	0.062	0.056	0.085
	-5	0.163	0.163	0.174	0.150	0.269
	-10	0.541	0.490	0.572	0.542	0.661
	-20	0.954	0.947	0.956	0.939	0.979
$\sqrt{0.7}$	0	0.057	0.053	0.052	0.066	0.087
	-5	0.489	0.508	0.497	0.450	0.606
	-10	0.890	0.897	0.892	0.874	0.939
	-20	0.999	0.999	0.999	0.998	1.000

Table 12: Size - MS: $p_{11} = 0.8$, $p_{22} = 0.7$

Φ_1	Θ_1	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
		$n = 50$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.057	0.062	0.057	0.049	0.085
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.079	0.079	0.071	0.079	0.153
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.066	0.064	0.055	0.055	0.113
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.050	0.048	0.052	0.063	0.079
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.063	0.072	0.071	0.067	0.112
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.046	0.055	0.047	0.050	0.087
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.270	0.311	0.242	0.300	0.397
		$n = 100$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.051	0.043	0.051	0.047	0.063
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.072	0.081	0.072	0.078	0.113
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.071	0.065	0.075	0.080	0.095
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.053	0.043	0.045	0.043	0.060
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.064	0.064	0.059	0.065	0.076
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.065	0.067	0.058	0.065	0.080
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.250	0.265	0.246	0.281	0.309

Table 13: Power - MS: $p_{11} = 0.8$, $p_{22} = 0.7$

dynamics	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
	-5	0.359	0.356	0.319	0.363	0.484
$\Phi_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.699	0.718	0.717	0.739	0.850
	-20	0.953	0.982	0.958	0.979	0.990
	-5	0.254	0.276	0.251	0.255	0.374
$\Theta_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.663	0.672	0.623	0.649	0.770
	-20	0.977	0.985	0.977	0.978	0.991
			$n = 100$			
	-5	0.443	0.374	0.443	0.414	0.473
$\Phi_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.833	0.839	0.831	0.851	0.875
	-20	0.994	0.998	0.997	0.996	0.999
	-5	0.250	0.234	0.262	0.240	0.325
$\Theta_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.690	0.730	0.691	0.725	0.757
	-20	0.994	0.990	0.994	0.994	0.997

Table 14: $\Phi_1 = \Theta_1 = 0$ - MS: $p_{11} = 0.3$, $p_{22} = 0.2$

r	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
0	0	0.108	0.135	0.125	0.150	0.252
	-5	0.388	0.517	0.352	0.471	0.652
	-10	0.725	0.820	0.754	0.830	0.882
	-20	0.954	0.972	0.952	0.971	0.985
$\sqrt{0.3}$	0	0.122	0.136	0.124	0.137	0.260
	-5	0.354	0.402	0.356	0.441	0.620
	-10	0.697	0.795	0.671	0.801	0.894
	-20	0.958	0.974	0.942	0.976	0.989
$\sqrt{0.7}$	0	0.104	0.120	0.104	0.127	0.237
	-5	0.393	0.440	0.400	0.423	0.718
	-10	0.871	0.915	0.865	0.920	0.973
	-20	0.976	0.995	0.973	0.995	0.997
			$n = 100$			
0	0	0.121	0.125	0.101	0.133	0.168
	-5	0.344	0.343	0.376	0.361	0.449
	-10	0.609	0.609	0.597	0.602	0.697
	-20	0.922	0.933	0.907	0.920	0.975
$\sqrt{0.3}$	0	0.095	0.101	0.099	0.098	0.155
	-5	0.332	0.298	0.344	0.302	0.449
	-10	0.635	0.617	0.647	0.604	0.771
	-20	0.959	0.958	0.961	0.964	0.989
$\sqrt{0.7}$	0	0.119	0.124	0.112	0.114	0.173
	-5	0.432	0.481	0.455	0.467	0.631
	-10	0.875	0.867	0.865	0.876	0.947
	-20	0.998	0.997	0.999	0.999	1.000

Table 15: Size - MS: $p_{11} = 0.3$, $p_{22} = 0.2$

Φ	Θ	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
		$n = 50$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.038	0.047	0.039	0.040	0.074
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.073	0.067	0.068	0.070	0.121
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.043	0.063	0.048	0.068	0.097
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.047	0.043	0.044	0.043	0.076
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.052	0.051	0.045	0.050	0.084
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.043	0.041	0.039	0.036	0.072
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.237	0.272	0.255	0.277	0.363
		$n = 100$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.047	0.041	0.049	0.043	0.058
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.073	0.078	0.067	0.079	0.107
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.061	0.055	0.070	0.061	0.080
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.053	0.045	0.043	0.044	0.059
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.043	0.043	0.043	0.044	0.054
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.049	0.053	0.047	0.046	0.068
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.224	0.254	0.232	0.245	0.267

Table 16: Power - MS: $p_{11} = 0.3$, $p_{22} = 0.2$

dynamics	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.299	0.336	0.324	0.300	0.403
	-10	0.703	0.724	0.669	0.711	0.808
	-20	0.977	0.987	0.966	0.985	0.993
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.271	0.266	0.259	0.209	0.332
	-10	0.597	0.608	0.580	0.626	0.730
	-20	0.972	0.980	0.972	0.979	0.988
			$n = 100$			
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.333	0.326	0.330	0.332	0.380
	-10	0.758	0.769	0.741	0.742	0.782
	-20	0.990	0.995	0.992	0.996	0.997
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.235	0.206	0.237	0.205	0.289
	-10	0.694	0.691	0.661	0.705	0.728
	-20	0.983	0.981	0.984	0.992	0.992

2.3 Non-normal distributions

The next modification concerns the distribution of the error terms. As the simultaneous representation is equivalent to the conditional/marginal representation, we now investigate some cases where the errors are not normal. Moreover, we also investigate the performance of the tests if the assumptions on the moments of the error terms are not met.

We consider the following three distributions.

1. Marginal centered $\chi^2(3)$ distribution with a gaussian copula. With this setting we can investigate skewness.
2. Marginal Student t distributions with 2 degrees of freedom with a gaussian copula. This will make the distribution have heavy tails; specifically the errors have infinite variance.
3. Bivariate t distribution with 4 degrees of freedom. Apart from the heavy tails (infinite fourth moment), we also do not use a gaussian copula anymore.

Results for the χ^2 errors are given in Table 17 to 19; results for the $t(2)$ errors are given in Table 20 to 22 and results for the $t(4)$ errors are given in Table 23 to 25. The results with the χ^2 errors are very similar to the normal errors. The close correspondence between the VAR and conditional/marginal representation is still present. With the infinite variance Student t errors there is some undersize for all the bootstrap tests. Apart from this, the tests still perform well, and there is also no notable difference between the VAR and conditional/marginal representations. The performance of the tests with Student t errors with 4 degrees of freedom is again very similar to the performance with normal errors.

Table 17: No serial correlation - $\chi^2(3)$ errors

r	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
0	0	0.069	0.059	0.059	0.069	0.092
	-5	0.086	0.077	0.070	0.098	0.156
	-10	0.291	0.293	0.264	0.320	0.439
	-20	0.800	0.821	0.831	0.819	0.919
$\sqrt{0.3}$	0	0.048	0.042	0.051	0.050	0.081
	-5	0.146	0.174	0.116	0.174	0.234
	-10	0.457	0.506	0.519	0.498	0.615
	-20	0.928	0.931	0.924	0.943	0.978
$\sqrt{0.7}$	0	0.047	0.050	0.049	0.050	0.081
	-5	0.443	0.456	0.460	0.479	0.593
	-10	0.871	0.896	0.884	0.898	0.933
	-20	0.992	0.997	0.993	0.999	0.999
			$n = 100$			
0	0	0.056	0.056	0.051	0.056	0.071
	-5	0.133	0.116	0.132	0.116	0.155
	-10	0.312	0.304	0.304	0.313	0.361
	-20	0.899	0.862	0.859	0.862	0.901
$\sqrt{0.3}$	0	0.056	0.050	0.058	0.056	0.064
	-5	0.191	0.187	0.167	0.187	0.218
	-10	0.474	0.471	0.481	0.486	0.593
	-20	0.971	0.959	0.937	0.953	0.978
$\sqrt{0.7}$	0	0.061	0.068	0.052	0.059	0.086
	-5	0.513	0.492	0.513	0.542	0.568
	-10	0.895	0.884	0.893	0.896	0.917
	-20	0.999	1.000	1.000	1.000	1.000

Table 18: Size - $\chi^2(3)$ errors

Φ	Θ	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
		$n = 50$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.063	0.063	0.065	0.070	0.107
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.056	0.056	0.047	0.059	0.178
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.066	0.058	0.052	0.060	0.204
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.052	0.049	0.059	0.058	0.098
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.047	0.043	0.034	0.044	0.179
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.072	0.071	0.073	0.073	0.178
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.439	0.615	0.406	0.606	0.671
		$n = 100$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.077	0.061	0.064	0.066	0.084
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.045	0.047	0.049	0.039	0.091
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.053	0.048	0.043	0.042	0.098
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.064	0.072	0.059	0.062	0.080
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.039	0.051	0.050	0.057	0.096
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.065	0.066	0.068	0.070	0.138
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.475	0.545	0.499	0.535	0.619

Table 19: Power - $\chi^2(3)$ errors

dynamics	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
		$n = 50$				
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.559	0.504	0.559	0.497	0.823
	-10	0.791	0.778	0.781	0.798	0.947
	-20	0.836	0.936	0.871	0.936	0.987
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.339	0.249	0.334	0.300	0.585
	-10	0.719	0.608	0.723	0.642	0.908
	-20	0.921	0.962	0.934	0.963	0.992
		$n = 100$				
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.892	0.881	0.864	0.879	0.942
	-10	0.983	0.989	0.981	0.986	0.997
	-20	1.000	1.000	1.000	1.000	1.000
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.467	0.497	0.480	0.477	0.688
	-10	0.894	0.885	0.895	0.862	0.960
	-20	0.997	0.997	0.997	0.997	0.999

Table 20: No serial correlation - $t(2)$ errors

r	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n=50$			
0	0	0.032	0.032	0.029	0.037	0.059
	-5	0.087	0.055	0.063	0.079	0.149
	-10	0.211	0.168	0.222	0.165	0.385
	-20	0.840	0.762	0.840	0.775	0.935
$\sqrt{0.3}$	0	0.038	0.042	0.042	0.041	0.084
	-5	0.120	0.105	0.120	0.122	0.221
	-10	0.406	0.347	0.401	0.366	0.570
	-20	0.914	0.870	0.926	0.909	0.962
$\sqrt{0.7}$	0	0.025	0.036	0.021	0.039	0.074
	-5	0.347	0.331	0.370	0.371	0.534
	-10	0.768	0.801	0.758	0.795	0.889
	-20	0.978	0.982	0.972	0.980	0.992
			$n=100$			
0	0	0.029	0.029	0.036	0.029	0.060
	-5	0.109	0.067	0.074	0.078	0.130
	-10	0.202	0.270	0.251	0.219	0.338
	-20	0.843	0.838	0.875	0.834	0.913
$\sqrt{0.3}$	0	0.031	0.033	0.034	0.037	0.058
	-5	0.118	0.130	0.113	0.128	0.187
	-10	0.388	0.393	0.314	0.380	0.502
	-20	0.906	0.921	0.914	0.915	0.956
$\sqrt{0.7}$	0	0.035	0.027	0.025	0.026	0.058
	-5	0.310	0.311	0.319	0.311	0.469
	-10	0.738	0.732	0.763	0.725	0.855
	-20	0.988	0.985	0.988	0.987	0.993

Table 21: Size - $t(2)$ errors

Φ	Θ	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
		$n = 50$				
$\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}$	$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	0.047	0.047	0.052	0.045	0.088
$\begin{bmatrix}0.8 & 0\\ 0 & 0.8\end{bmatrix}$	$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	0.039	0.036	0.042	0.041	0.156
$\begin{bmatrix}0.2 & 0.5\\ 0.5 & 0.2\end{bmatrix}$	$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	0.040	0.057	0.056	0.045	0.177
$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}$	0.052	0.050	0.047	0.052	0.099
$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}0.8 & 0\\ 0 & 0.8\end{bmatrix}$	0.042	0.039	0.034	0.042	0.174
$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}0.2 & 0.5\\ 0.5 & 0.2\end{bmatrix}$	0.049	0.043	0.046	0.064	0.164
$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}-0.8 & 0\\ 0 & -0.8\end{bmatrix}$	0.443	0.613	0.477	0.597	0.686
		$n = 100$				
$\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}$	$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	0.056	0.039	0.049	0.053	0.080
$\begin{bmatrix}0.8 & 0\\ 0 & 0.8\end{bmatrix}$	$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	0.050	0.043	0.043	0.054	0.088
$\begin{bmatrix}0.2 & 0.5\\ 0.5 & 0.2\end{bmatrix}$	$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	0.045	0.047	0.047	0.043	0.100
$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}$	0.043	0.049	0.052	0.043	0.075
$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}0.8 & 0\\ 0 & 0.8\end{bmatrix}$	0.035	0.034	0.034	0.040	0.100
$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}0.2 & 0.5\\ 0.5 & 0.2\end{bmatrix}$	0.057	0.071	0.066	0.067	0.123
$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}-0.8 & 0\\ 0 & -0.8\end{bmatrix}$	0.464	0.505	0.465	0.523	0.597

Table 22: Power - $t(2)$ errors

dynamics	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
	-5	0.489	0.499	0.485	0.508	0.764
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.627	0.733	0.701	0.745	0.902
	-20	0.812	0.870	0.825	0.862	0.961
	-5	0.354	0.301	0.332	0.310	0.589
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.716	0.630	0.691	0.624	0.897
	-20	0.888	0.911	0.871	0.909	0.982
			$n = 100$			
	-5	0.819	0.822	0.805	0.808	0.892
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.942	0.959	0.939	0.935	0.977
	-20	0.982	0.985	0.980	0.985	0.993
	-5	0.423	0.458	0.380	0.399	0.621
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-10	0.812	0.835	0.795	0.765	0.919
	-20	0.983	0.985	0.987	0.987	0.995

Table 23: No serial correlation - $t(4)$ errors

r	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
0	0	0.041	0.051	0.057	0.042	0.077
	-5	0.106	0.101	0.094	0.109	0.163
	-10	0.295	0.280	0.305	0.315	0.420
	-20	0.867	0.875	0.895	0.868	0.938
$\sqrt{0.3}$	0	0.044	0.045	0.041	0.035	0.069
	-5	0.170	0.180	0.172	0.196	0.257
	-10	0.477	0.485	0.469	0.498	0.624
	-20	0.949	0.958	0.951	0.949	0.978
$\sqrt{0.7}$	0	0.043	0.048	0.043	0.043	0.070
	-5	0.499	0.507	0.462	0.513	0.628
	-10	0.878	0.901	0.900	0.886	0.939
	-20	0.985	0.995	0.985	0.995	0.996
			$n = 100$			
0	0	0.059	0.056	0.061	0.052	0.071
	-5	0.109	0.082	0.116	0.114	0.130
	-10	0.349	0.338	0.364	0.304	0.380
	-20	0.897	0.876	0.884	0.902	0.917
$\sqrt{0.3}$	0	0.044	0.051	0.049	0.048	0.063
	-5	0.177	0.186	0.197	0.190	0.237
	-10	0.578	0.551	0.561	0.484	0.603
	-20	0.971	0.968	0.965	0.961	0.974
$\sqrt{0.7}$	0	0.051	0.051	0.047	0.043	0.060
	-5	0.590	0.563	0.584	0.577	0.611
	-10	0.918	0.927	0.925	0.919	0.936
	-20	1.000	1.000	1.000	1.000	1.000

Table 24: Size - $t(4)$ errors

Φ	Θ	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
		$n = 50$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.055	0.049	0.056	0.054	0.088
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.054	0.041	0.052	0.042	0.176
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.051	0.053	0.042	0.059	0.196
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.076	0.067	0.074	0.067	0.107
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.042	0.041	0.049	0.039	0.187
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.064	0.077	0.063	0.055	0.183
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.422	0.613	0.448	0.610	0.681
		$n = 100$				
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.053	0.065	0.066	0.058	0.080
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.047	0.055	0.053	0.053	0.090
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.056	0.044	0.061	0.056	0.098
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.071	0.072	0.059	0.070	0.076
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.047	0.047	0.050	0.052	0.110
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.067	0.059	0.059	0.067	0.141
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.445	0.529	0.446	0.537	0.595

Table 25: Power - $t(4)$ errors

dynamics	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.578	0.562	0.608	0.597	0.828
	-10	0.773	0.782	0.766	0.791	0.943
	-20	0.819	0.946	0.804	0.946	0.987
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.434	0.339	0.440	0.357	0.650
	-10	0.712	0.667	0.761	0.686	0.906
	-20	0.928	0.969	0.931	0.967	0.994
			$n = 100$			
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.871	0.890	0.872	0.886	0.943
	-10	0.986	0.985	0.986	0.988	0.995
	-20	0.997	0.999	0.997	0.999	0.999
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.514	0.519	0.531	0.496	0.701
	-10	0.901	0.876	0.889	0.875	0.959
	-20	0.991	0.992	0.993	0.992	0.998

2.4 Lag length selection by AIC

The final modification is unrelated to the simulation DGP, but concerns the way the lag lengths are selected. In the paper we use BIC, here we will use AIC as an alternative. In general AIC will select higher lag lengths, and we wish to investigate what effect this has on the tests.

Results are given in Table 26 to 28. We see that the bootstrap tests still have reasonably good size, although they are now somewhat undersized. There is a notable improvement of the size with the large negative VMA term. Surprisingly, the size properties of the asymptotic test actually worsen: there is even more oversize. The power properties are clearly worse for all tests than using BIC, especially for the bootstrap tests and $n = 50$.

This is a clear argument for using BIC instead of AIC, especially since there is no size improvement (apart from the negative VMA case). Another argument in favor of BIC is that the bootstrap tests estimated under the alternative hypothesis become less stable for longer lag lengths. That is, nonstationary processes are more often estimated from which one cannot resample. Estimation under the null is very stable and its stability is also not affected by AIC.

Table 26: No serial correlation - AIC

r	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
0	0	0.057	0.039	0.055	0.045	0.166
	-5	0.048	0.061	0.065	0.060	0.234
	-10	0.098	0.089	0.114	0.089	0.433
	-20	0.403	0.425	0.403	0.446	0.842
$\sqrt{0.3}$	0	0.036	0.030	0.025	0.036	0.139
	-5	0.079	0.098	0.077	0.101	0.338
	-10	0.221	0.222	0.198	0.221	0.623
	-20	0.569	0.710	0.523	0.694	0.903
$\sqrt{0.7}$	0	0.058	0.050	0.048	0.055	0.153
	-5	0.277	0.273	0.214	0.265	0.623
	-10	0.553	0.608	0.505	0.671	0.883
	-20	0.772	0.865	0.727	0.868	0.946
			$n = 100$			
0	0	0.049	0.045	0.044	0.051	0.088
	-5	0.094	0.093	0.115	0.117	0.172
	-10	0.213	0.240	0.200	0.215	0.385
	-20	0.679	0.700	0.688	0.700	0.848
$\sqrt{0.3}$	0	0.055	0.044	0.043	0.047	0.083
	-5	0.156	0.150	0.157	0.150	0.253
	-10	0.426	0.407	0.407	0.383	0.601
	-20	0.878	0.906	0.874	0.876	0.947
$\sqrt{0.7}$	0	0.048	0.043	0.042	0.049	0.081
	-5	0.446	0.527	0.451	0.423	0.593
	-10	0.826	0.859	0.803	0.867	0.918
	-20	0.956	0.972	0.963	0.968	0.980

Table 27: Size - AIC

Φ	Θ	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.036	0.035	0.036	0.033	0.168
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.029	0.033	0.033	0.033	0.258
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.035	0.036	0.033	0.035	0.210
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.041	0.036	0.035	0.036	0.160
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.025	0.021	0.025	0.029	0.206
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.040	0.032	0.046	0.039	0.221
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.229	0.274	0.227	0.269	0.423
			$n = 100$			
$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.054	0.050	0.051	0.049	0.102
$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.051	0.040	0.049	0.059	0.129
$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.051	0.045	0.044	0.054	0.113
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$	0.057	0.058	0.053	0.067	0.098
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$	0.039	0.036	0.042	0.051	0.119
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	0.051	0.056	0.051	0.051	0.123
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$	0.238	0.257	0.248	0.251	0.348

Table 28: Power - AIC

dynamics	c	$T^*_{v,n}$	$T^*_{v,a}$	$T^*_{c,n}$	$T^*_{c,a}$	T
			$n = 50$			
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.368	0.452	0.396	0.446	0.858
	-10	0.541	0.674	0.506	0.679	0.917
	-20	0.628	0.785	0.631	0.787	0.933
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.178	0.201	0.179	0.194	0.653
	-10	0.423	0.481	0.415	0.472	0.870
	-20	0.617	0.760	0.591	0.738	0.923
			$n = 100$			
$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.847	0.878	0.808	0.868	0.947
	-10	0.935	0.964	0.926	0.965	0.982
	-20	0.946	0.974	0.938	0.976	0.988
$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$	-5	0.439	0.490	0.449	0.490	0.731
	-10	0.823	0.853	0.797	0.822	0.941
	-20	0.917	0.950	0.917	0.948	0.978

3 Deterministic Trends

Finally we add a small extension to the simulations with deterministic trends. In the paper we recommend not to include deterministic components in Step 3 of the bootstrap algorithm, as the tests are asymptotically similar. In the following simulation study we do add deterministic components (that we estimated in Step 1).

The results are given in Table 29. To be able to have a direct comparison with the original simulations using the WSB (as given in Table 4), we use the same seed for both sets of simulations. As can be seen from the two tables, the rejection frequencies for both experiments are exactly the same. This indicates that the tests are not only similar asymptotically, but also in finite samples. Therefore it is unnecessary to include deterministic components in Step 3.

References

Giacomini, R., D. N. Politis, and H. White (2007). A Warp-Speed method for conducting Monte Carlo experiments involving bootstrap estimators. Working paper.

Table 29: Size and power for tests with deterministic trends

μ_1 τ_1 / μ_2 τ_2	$D_t^{(r)}$	$T^*_{v,n}$	$T^*_{v,a}$	T_{as}	$T^*_{v,n}$	$T^*_{v,a}$	T_{as}	$T^*_{v,n}$	$T^*_{v,a}$	T_{as}
		$\Phi = \Theta = 0$			$\Phi = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$			$\Theta = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$		
					$c = 0$					
	$D_t^r = 1$	0.046	0.056	0.110	0.057	0.066	0.282	0.051	0.053	0.234
0 0	$D_t = 1$	0.051	0.054	0.093	0.048	0.061	0.231	0.059	0.068	0.230
0 0	$D_t^r = t$	0.032	0.035	0.125	0.045	0.070	0.358	0.034	0.069	0.330
	$D_t' = 1, t$	0.056	0.056	0.129	0.045	0.057	0.338	0.061	0.088	0.333
	$D_t^r = 1$	0.055	0.052	0.105	0.047	0.064	0.287	0.060	0.059	0.245
1 0	$D_t = 1$	0.056	0.048	0.091	0.051	0.059	0.260	0.051	0.069	0.231
1 0	$D_t^r = t$	0.046	0.048	0.130	0.035	0.068	0.372	0.046	0.070	0.329
	$D_t' = 1, t$	0.047	0.050	0.133	0.053	0.057	0.339	0.036	0.053	0.326
1 0	$D_t^r = t$	0.049	0.047	0.135	0.029	0.069	0.367	0.039	0.061	0.334
1 1	$D_t' = 1, t$	0.048	0.048	0.136	0.035	0.068	0.346	0.040	0.085	0.343
1 1	$D_t^r = t$	0.050	0.049	0.143	0.044	0.069	0.371	0.035	0.054	0.348
1 1	$D_t' = 1, t$	0.052	0.044	0.123	0.032	0.051	0.350	0.045	0.048	0.348
					$c = -10$					
	$D_t^r = 1$	0.235	0.293	0.434	0.574	0.579	0.894	0.584	0.370	0.802
0 0	$D_t = 1$	0.303	0.301	0.454	0.580	0.591	0.880	0.469	0.371	0.798
0 0	$D_t^r = t$	0.141	0.143	0.367	0.382	0.306	0.835	0.285	0.135	0.709
	$D_t' = 1, t$	0.169	0.171	0.385	0.378	0.278	0.836	0.303	0.186	0.716
	$D_t^r = 1$	0.236	0.217	0.427	0.585	0.560	0.900	0.514	0.463	0.806
1 0	$D_t = 1$	0.255	0.254	0.443	0.544	0.555	0.881	0.513	0.434	0.799
1 0	$D_t^r = t$	0.159	0.136	0.365	0.431	0.307	0.833	0.336	0.127	0.734
	$D_t' = 1, t$	0.190	0.171	0.388	0.370	0.295	0.830	0.294	0.196	0.718
1 0	$D_t^r = t$	0.163	0.161	0.356	0.423	0.304	0.846	0.312	0.217	0.737
1 1	$D_t' = 1, t$	0.173	0.177	0.400	0.364	0.234	0.818	0.269	0.178	0.687
1 1	$D_t^r = t$	0.173	0.171	0.368	0.374	0.287	0.853	0.324	0.135	0.707
1 1	$D_t' = 1, t$	0.154	0.167	0.401	0.393	0.282	0.809	0.271	0.194	0.728